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Detecting Latent Clinical Taxa, IX:

A Monte Carlo Method for

Testing Taxometric Theories[1]

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and

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Report No. PR-74-7 October 1974

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Appendix A

Subroutine for Random Number Generator

1. Instructions for use of the subroutine

Multivariate Bernoulli Random
Number Generator

Subroutine DEPBNG

a) CALL DEPBNG(IFLAG, Y, SIGMA, T, M, N, SEED, P)

IFLAG: an integer input parameter

If IFLAG < 0, a value of SEED and the transformed covariance matrix are produced,

If IFLAG = 0, the transformed covariance matrix is produced,

If IFLAG > 0, the array Y is filled with N · M random numbers; SEED is not specified by the user.

(N: number of observations)

(M: number of variables)

There are at least two calls needed for this routine. The first with IFLAG < 0 initializes the routine and the second or any later calls with IFLAG > 0 generates random vectors.

Y: a real output array, dimensioned at least N · M which is filled with N random vectors on a call with IFLAG > 0.

SIGMA: an input-output array, dimensioned at least M · M. When IFLAG < 0, sigma contains the lower triangle of the correlation matrix for the multivariate normal random variables. However, when IFLAG ≥ 0, It contains a transformation of the correlation matrix. Between the initial call and the following calls, SIGMA should not be changed.

T: a scratch array with dimension N

M: an integer input parameter which is the number of Bernoulli random variables

N: an integer input parameter which is the sample size

SEED: an input-output variable which keeps the current location in the sequence of numbers being generated. SEED is changed by the subroutine during each call after the initial one. It should not be changed by the user after the first call, but can be specified at the initial call if desired.

P: an input array, dimensioned at least M, which contains the Bernoulli variable parameters. $0 < P_i < 1$ for $i = 1, 2, \ldots, M$.

b) Space required: CM = 3700_8

c) Timing on CDC 6600 computer:

5 variables, 1000 observations: 981 milliseconds

5 variables, 5000 observations: 4897 milliseconds

2. Listing of the subroutine

```
                SUBROUTINE DEPBNG(IFLAG,Y,C,G,M,N,SEED,P)

C****S. TYAN---MULTIVARIATE BERNOULLI RANDOM NUMBER GENERATOR
C    IFLAG = NEGATIVE INTEGER = GENERATE THE SEED AND
C                               TRANSFORM THE CORRECTION MATRIX
C              = 0 = TRANSFORM THE CORRELATION MATRIX ONLY
C              =POSITIVE INTEGER=GENERATE THE RANDOM NUMBERS
C    C = THE CORRELATION MATRIX OF THE SIZE M*M, THE
C         DIAGONAL ELEMENTS SHOULD BE ONES
C    P = THE VECTOR OF THE BERNOULLI VARIABLE PARAMETERS
C    Y = THE OUTPUT MATRIX OF SIZE N*M FOR THE RANDOM
C          VECTORS
C    M = THE NUMBER OF VARIABLES
C    N = SAMPLE SIZE
C****G = SCRATCH ARRAY WITH TEMPORARY STORAGE OF SIZE AT LEAST N
     DIMENSION Y(N,M),C(M,M),G(N),P(M)
     IF(IFLAG)100,200,300
100 CALL NORMAL(IFLAG,Y,SEED)
C********PERFORM A DIAGONAL FACTORING OF C
200 ER=6HM.LT.1
     IF(M.LT.1)GO TO 250
     EPS=C(1,1)
     DO 210 I=1,M
210 IF(C(I,I).GT.EPS)EPS=C(I,I)
     EPS=EPS*1.0E-10 $ ER=7HC-NEG.
     DO 220 I=1,M
     T=C(I,I) $ IF(T.GT.EPS)GO TO 222
     IF(T.LT.-EPS)GO TO 250
```

```
     DO 221 J=1,M
221 C(J,I)=0.
     GO TO 220
222 T=SQRT(T)
     C(I,I)=T $ IF(I.GE.M)RETURN
     T=1./T $ JJ=I+1
     DO 225 J=JJ,M
     C(J,I)=C(J,I)*T
225 C(I,J)=0.
     DO 228 J=JJ,M
     T=C(J,I)
     DO 228 K=J,M
228 C(K,J)=C(K,J)-C(K,I)*T
220 CONTINUE
     RETURN
C**********ERROR PRINT
250 PRINT l,ER
  1 FORMAT(lX99(1H*)/12H0ERROR,A8)
     STOP
300 II=M*N
     CALL NORMAL(II,Y,SEED)
     DO 350 I=1,M
     DO 352 K=1,N
352 G(K)=0.
     IM=JM=M-I+1 $ II=0
     DO 355 J=1,IM
```

```
      DO 354 K=1,N
      II= II+1
354 G(K)=G(K)+Y(II)*C(JM)
355 JM=JM+M
      II=(IM-1)*N
      DO 356 K=1,N
      II=II+1
356 Y(II)=G(K)
350 CONTINUE
      DO 360 JJ=1,N
      J=JJ
      DO 360 NO=1,M
      X=CDFN(Y(J)) $ Y(J)=0.
      IF(X.LE.P(NO))Y(J)=1.
360 J=J+N
      RETURN
      END
```

Subroutine NORMAL generates pseudo-normal deviates with zero mean and unit variance. The arguments IFLAG, Y, and SEED are defined above. Subroutine CDFN calculates the cumulative normal density for a given standard deviant which is the argument.

APPENDIX B

Tables giving ρ_{ij} as a function of

p_i, p_j, and ϕ_{ij}

$p_i = .10$	$p_j = .10$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.010	–.000	.000
.011	.009	.025
.012	.018	.050
.012	.027	.075
.013	.037	.100
.014	.047	.125
.015	.058	.150
.016	.068	.175
.017	.080	.200
.018	.091	.225
.019	.103	.250
.020	.116	.275
.022	.129	.300
.023	.142	.325
.024	.156	.350
.025	.170	.375
.027	.185	.400
.028	.200	.425
.029	.215	.450
.031	.232	.475
.032	.248	.500
.034	.266	.525
.036	.284	.550
.037	.302	.575
.039	.322	.600
.041	.342	.625
.043	.363	.650
.045	.385	.675
.047	.408	.700
.049	.432	.725
.051	.458	.750
.054	.485	.775
.056	.513	.800
.059	.544	.825
.062	.577	.850
.065	.613	.875
.069	.654	.900
.073	.700	.925
.078	.760	.950
.090	.884	.975

$p_i = .10$ $p_j = .20$

p_{ij}	ϕ_{ij}	ρ_{ij}
.020	–.000	.000
.021	.010	.025
.023	.021	.050
.024	.032	.075
.025	.043	.100
.027	.054	.125
.028	.066	.150
.029	.078	.175
.031	.090	.200
.032	.103	.225
.034	.116	.250
.035	.129	.275
.037	.143	.300
.039	.156	.325
.040	.170	.350
.042	.185	.375
.044	.200	.400
.046	.215	.425
.048	.230	.450
.049	.246	.475
.051	.262	.500
.053	.279	.525
.055	.296	.550
.058	.313	.575
.060	.331	.600
.062	.349	.625
.064	.368	.650
.067	.388	.675
.069	.408	.700
.071	.428	.725
.074	.450	.750
.077	.472	.775
.079	.495	.800
.082	.518	.825
.085	.543	.850
.088	.568	.875
.091	.593	.900
.094	.617	.925
.096	.636	.950
.093	.608	.975

$p_i = .10$	$p_j = .30$	
	ϕ_{ij}	ρ_{ij}
.030	−.000	.000
.032	.011	.025
.033	.022	.050
.035	.034	.075
.036	.046	.100
.038	.058	.125
.040	.070	.150
.041	.082	.175
.043	.094	.200
.045	.107	.225
.046	.120	.250
.048	.133	.275
.050	.146	.300
.052	.159	.325
.054	.172	.350
.056	.186	.375
.057	.200	.400
.059	.214	.425
.061	.228	.450
.063	.242	.475
.065	.257	.500
.067	.271	.525
.069	.286	.550
.071	.301	.575
.073	.316	.600
.076	.331	.625
.078	.347	.650
.080	.362	.675
.082	.378	.700
.084	.393	.725
.086	.409	.750
.088	.424	.775
.090	.438	.800
.092	.454	.825
.094	.468	.850
.096	.481	.875
.098	.491	.900
.098	.497	.925
.097	.491	.950
.092	.453	.975

$p_i = .10$ $p_j = .40$

p_{ij}	ϕ_{ij}	ρ_{ij}
.040	–.000	.000
.042	.011	.025
.043	.023	.050
.045	.035	.075
.047	.047	.100
.049	.059	.125
.050	.071	.150
.052	.083	.175
.054	.095	.200
.056	.107	.225
.058	.119	.250
.059	.131	.275
.061	.144	.300
.063	.156	.325
.065	.168	.350
.067	.181	.375
.068	.193	.400
.070	.206	.425
.072	.218	.450
.074	.231	.475
.076	.243	.500
.078	.256	.525
.079	.268	.575
.081	.280	.600
.083	.292	.625
.085	.304	.650
.086	.316	.675
.088	.328	.700
.090	.339	.725
.091	.350	.750
.093	.361	.775
.094	.371	.725
.096	.380	.800
.097	.388	.825
.098	.395	.850
.099	.401	.875

$p_i = .10$ $p_j = .50$

p_{ij}	ϕ_{ij}	ρ_{ij}
.050	−.000	.000
.052	.012	.025
.053	.023	.050
.055	.035	.075
.057	.046	.100
.059	.058	.125
.060	.070	.150
.062	.081	.175
.064	.093	.200
.066	.104	.225
.067	.116	.250
.069	.127	.275
.071	.138	.300
.072	.149	.325
.074	.161	.350
.076	.172	.375
.077	.183	.400
.079	.193	.425
.081	.204	.450
.082	.215	.475
.084	.225	.500
.085	.235	.525
.087	.245	.550
.088	.255	.575
.090	.264	.600
.091	.273	.625
.092	.282	.650
.093	.290	.675
.095	.298	.700
.096	.305	.725
.097	.312	.750
.098	.317	.775
.098	.323	.800
.099	.327	.825

$p_i = .10$	$p_j = .60$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.060	–.000	.000
.062	.011	.025
.063	.023	.050
.065	.034	.075
.067	.045	.100
.068	.056	.125
.070	.067	.150
.071	.078	.175
.073	.089	.200
.075	.099	.225
.076	.110	.250
.078	.120	.275
.079	.130	.300
.081	.140	.325
.082	.149	.350
.083	.159	.375
.085	.168	.400
.086	.177	.425
.087	.186	.450
.089	.194	.475
.090	.203	.500
.091	.211	.525
.092	.218	.550
.093	.225	.575
.094	.232	.600
.095	.239	.625
.096	.245	.650
.097	.250	.675
.097	.255	.700
.098	.259	.725
.099	.263	.750
.099	.267	.775

$p_i = .10$ $p_j = .70$

p_{ij}	ϕ_{ij}	ρ_{ij}
.070	−.000	.000
.071	.011	.025
.073	.022	.050
.074	.032	.075
.076	.043	.100
.077	.053	.125
.079	.063	.150
.080	.073	.175
.081	.082	.200
.083	.092	.225
.084	.101	.250
.085	.109	.275
.086	.118	.300
.087	.126	.325
.088	.134	.350
.090	.142	.375
.091	.150	.400
.092	.157	.425
.093	.164	.450
.093	.170	.475
.094	.176	.500
.095	.182	.525
.096	.188	.550
.096	.193	.575
.097	.197	.600
.098	.201	.625
.098	.205	.650
.099	.208	.675
.099	.211	.700
.099	.213	.725

$p_i = .10$	$p_j = .80$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.080	–.000	.000
.081	.010	.025
.082	.020	.050
.084	.029	.075
.085	.038	.100
.086	.047	.125
.087	.056	.150
.088	.065	.175
.089	.073	.200
.090	.080	.225
.091	.088	.250
.091	.095	.275
.092	.102	.300
.093	.108	.325
.094	.115	.350
.094	.121	.375
.095	.126	.400
.096	.131	.425
.096	.136	.450
.097	.141	.475
.097	.145	.500
.098	.148	.525
.098	.152	.550
.099	.155	.575
.099	.157	.600
.099	.160	.625
.099	.161	.650

$p_i = .10$ $p_j = .90$

p_{ij}	ϕ_{ij}	ρ_{ij}
.090	−.000	.000
.091	.008	.025
.091	.016	.050
.092	.024	.075
.093	.031	.100
.093	.038	.125
.094	.045	.150
.095	.051	.175
.095	.057	.200
.096	.063	.225
.096	.068	.250
.097	.073	.275
.097	.078	.300
.097	.082	.325
.098	.086	.350
.098	.090	.375
.098	.093	.400
.099	.096	.425
.099	.098	.450
.099	.101	.475
.099	.102	.500
.099	.104	.525
.099	.105	.550

$p_i = .20$	$p_j = .20$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.040	–.000	.000
.042	.012	.025
.044	.025	.050
.046	.038	.075
.048	.051	.100
.050	.064	.125
.052	.077	.150
.055	.091	.175
.057	.105	.200
.059	.119	.225
.061	.133	.250
.064	.148	.275
.066	.163	.300
.069	.178	.325
.071	.194	.350
.074	.210	.375
.076	.226	.400
.079	.242	.425
.081	.259	.450
.084	.277	.475
.087	.294	.500
.090	.312	.525
.093	.331	.550
.096	.350	.575
.099	.370	.600
.102	.390	.625
.106	.411	.650
.109	.433	.675
.113	.455	.700
.117	.479	.725
.121	.503	.750
.125	.529	.775
.129	.556	.800
.134	.585	.825
.139	.616	.850
.144	.650	.875
.150	.688	.900
.157	.734	.925
.168	.800	.950
.197	.982	.975

$p_i = .20$ $p_j = .30$

p_{ij}	ϕ_{ij}	ρ_{ij}
.060	−.000	.000
.062	.013	.025
.065	.027	.050
.067	.040	.075
.070	.054	.100
.072	.068	.125
.075	.082	.150
.078	.096	.175
.080	.111	.200
.083	.126	.225
.086	.140	.250
.088	.155	.275
.091	.171	.300
.094	.186	.325
.097	.202	.350
.100	.218	.375
.103	.234	.400
.106	.250	.425
.109	.267	.450
.112	.284	.475
.115	.301	.500
.118	.319	.525
.122	.337	.550
.125	.355	.575
.129	.374	.600
.132	.393	.625
.136	.413	.650
.139	.433	.675
.143	.454	.700
.147	.476	.725
.151	.498	.750
.156	.521	.775
.160	.545	.800
.165	.570	.825
.169	.597	.850
.174	.625	.875
.180	.654	.900
.186	.686	.925
.192	.719	.950
.191	.717	.975

$p_i = .20$ $p_j = .40$

p_{ij}	ϕ_{ij}	ρ_{ij}
.080	–.000	.000
.083	.014	.025
.085	.028	.050
.088	.042	.075
.091	.056	.100
.094	.070	.125
.096	.084	.150
.099	.098	.175
.102	.113	.200
.105	.127	.225
.108	.142	.250
.111	.157	.275
.114	.172	.300
.117	.187	.325
.120	.202	.350
.123	.217	.375
.126	.232	.400
.129	.248	.425
.132	.264	.450
.135	.279	.475
.138	.296	.500
.141	.312	.525
.144	.328	.550
.148	.345	.575
.151	.362	.600
.154	.379	.625
.158	.396	.650
.161	.414	.675
.165	.431	.700
.168	.449	.725
.172	.467	.750
.175	.486	.775
.179	.504	.800
.182	.523	.825
.186	.541	.850
.189	.558	.875
.192	.573	.900
.194	.581	.925
.191	.568	.950
.173	.475	.975

$p_i = .20$ $p_j = .50$

p_{ij}	ϕ_{ij}	ρ_{ij}
.100	–.000	.000
.103	.014	.025
.106	.028	.050
.108	.042	.075
.111	.056	.100
.114	.070	.125
.117	.084	.150
.120	.098	.175
.122	.112	.200
.125	.126	.225
.128	.140	.250
.131	.154	.275
.134	.168	.300
.136	.182	.325
.139	.196	.350
.142	.210	.375
.145	.225	.400
.148	.239	.425
.151	.253	.450
.153	.287	.475
.156	.282	.500
.159	.296	.525
.162	.310	.550
.165	.325	.575
.168	.339	.600
.171	.353	.625
.174	.368	.650
.176	.382	.675
.179	.396	.700
.182	.410	.725
.185	.423	.750
.187	.436	.775
.190	.449	.800
.192	.461	.825
.194	.471	.850
.196	.480	.875
.197	.484	.900
.196	.479	.925
.191	.454	.950
.177	.387	.975

$p_i = .20$ $p_j = .60$

p_{ij}	ϕ_{ij}	ρ_{ij}
.120	–.000	.000
.123	.014	.025
.125	.027	.050
.128	.041	.075
.131	.054	.100
.133	.068	.125
.136	.081	.150
.139	.095	.175
.141	.108	.200
.144	.121	.225
.146	.134	.250
.149	.147	.275
.151	.160	.300
.154	.173	.325
.156	.186	.350
.159	.199	.375
.161	.212	.400
.164	.224	.425
.166	.237	.450
.169	.249	.475
.171	.261	.500
.174	.273	.525
.176	.285	.550
.178	.297	.575
.180	.309	.600
.183	.320	.625
.185	.331	.650
.187	.341	.675
.189	.352	.700
.191	.361	.725
.193	.371	.750
.194	.379	.775
.196	.387	.800
.197	.394	.825
.198	.400	.850
.199	.404	.875

$p_i = .20$ $p_j = .70$

p_{ij}	ϕ_{ij}	ρ_{ij}
.140	−.000	.000
.142	.013	.025
.145	.026	.050
.147	.039	.075
.149	.052	.100
.152	.064	.125
.154	.077	.150
.156	.089	.175
.159	.101	.200
.161	.113	.225
.163	.125	.250
.165	.137	.275
.167	.148	.300
.169	.160	.325
.171	.171	.350
.173	.182	.375
.175	.193	.400
.177	.203	.425
.179	.214	.450
.181	.224	.475
.183	.234	.500
.185	.243	.525
.186	.253	.550
.188	.262	.575
.190	.270	.600
.191	.278	.625
.192	.286	.650
.194	.294	.675
.195	.300	.700
.196	.307	.725
.197	.312	.750
.198	.318	.775
.199	.323	.800

$p_i = .20$	$p_j = .80$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.160	–.000	.000
.162	.012	.025
.164	.024	.050
.166	.036	.075
.168	.047	.100
.169	.058	.125
.171	.069	.150
.173	.080	.175
.175	.091	.200
.175	.101	.225
.176	.111	.250
.179	.121	.275
.181	.131	.300
.182	.140	.325
.184	.149	.350
.185	.158	.375
.187	.167	.400
.188	.175	.425
.189	.183	.450
.190	.190	.475
.192	.197	.500
.193	.204	.525
.194	.211	.550
.195	.217	.575
.196	.223	.600
.196	.228	.625
.197	.233	.650
.198	.237	.675
.199	.241	.700
.199	.245	.725

$p_i = .20$ $p_j = .90$

p_{ij}	ϕ_{ij}	ρ_{ij}
.180	−.000	.000
.181	.010	.025
.182	.020	.050
.184	.030	.075
.185	.039	.100
.186	.048	.125
.187	.057	.150
.188	.065	.175
.189	.073	.200
.190	.081	.225
.191	.088	.250
.191	.096	.275
.192	.103	.300
.193	.109	.325
.194	.115	.350
.195	.121	.375
.195	.127	.400
.196	.132	.425
.196	.137	.450
.197	.141	.475
.197	.146	.500
.198	.149	.525
.198	.153	.550
.199	.156	.575
.199	.158	.600
.199	.161	.625

$p_i = .30$	$p_j = .30$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.090	–.000	.000
.093	.014	.025
.096	.029	.050
.099	.044	.075
.102	.058	.100
.105	.073	.125
.109	.088	.150
.112	.103	.175
.115	.119	.200
.118	.134	.225
.121	.150	.250
.125	.165	.275
.128	.181	.300
.131	.198	.325
.135	.214	.350
.138	.231	.375
.142	.247	.400
.146	.265	.425
.149	.282	.450
.153	.300	.475
.157	.318	.500
.161	.336	.525
.165	.355	.550
.169	.374	.575
.173	.394	.600
.177	.414	.625
.181	.435	.650
.186	.456	.675
.190	.478	.700
.195	.501	.725

$p_i = .30$ $p_j = .40$

p_{ij}	ϕ_{ij}	ρ_{ij}
.120	−.000	.000
.123	.015	.025
.127	.030	.050
.130	.045	.075
.134	.060	.100
.137	.075	.125
.140	.091	.150
.144	.106	.175
.147	.122	.200
.151	.137	.225
.154	.153	.250
.158	.169	.275
.162	.185	.300
.165	.201	.325
.169	.217	.350
.173	.234	.375
.176	.251	.400
.180	.268	.425
.184	.285	.450
.188	.302	.475
.192	.320	.500
.196	.338	.525
.200	.356	.550
.204	.375	.575
.208	.394	.600
.213	.413	.625
.217	.433	.650
.222	.454	.675
.227	.475	.700
.231	.496	.725
.236	.519	.750
.242	.542	.775
.247	.566	.800
.253	.592	.825
.259	.619	.850
.265	.647	.875
.272	.679	.900
.281	.716	.925
.291	.764	.950

$p_i = .30$	$p_j = .50$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.150	–.000	.000
.153	.015	.025
.157	.030	.050
.160	.045	.075
.164	.060	.100
.167	.076	.125
.171	.091	.150
.174	.106	.175
.178	.121	.200
.181	.137	.225
.185	.152	.250
.188	.168	.275
.192	.183	.300
.196	.199	.325
.199	.215	.350
.203	.231	.375
.207	.247	.400
.210	.263	.425
.214	.279	.450
.218	.296	.475
.222	.312	.500
.225	.329	.525
.229	.346	.550
.233	.363	.575
.237	.381	.600
.241	.398	.625
.245	.416	.650
.249	.434	.675
.254	.453	.700
.258	.471	.725
.262	.490	.750
.267	.509	.775
.271	.529	.800
.276	.548	.825
.280	.568	.850
.284	.586	.875
.288	.603	.900
.290	.611	.925
.285	.590	.950
.252	.445	.975

$p_i = .30$ $p_j = .60$

p_{ij}	ϕ_{ij}	ρ_{ij}
.180	–.000	.000
.183	.015	.025
.187	.030	.050
.190	.044	.075
.193	.059	.100
.197	.074	.125
.200	.089	.150
.203	.104	.175
.207	.118	.200
.210	.133	.225
.213	.148	.250
.216	.163	.275
.220	.177	.300
.223	.192	.325
.226	.207	.350
.230	.222	.375
.233	.236	.400
.236	.251	.425
.240	.266	.450
.243	.281	.475
.246	.296	.500
.250	.311	.525
.253	.326	.550
.257	.341	.575
.260	.356	.600
.263	.371	.625
.267	.386	.650
.270	.401	.675
.273	.416	.700
.277	.430	.725
.280	.445	.750
.283	.459	.775
.286	.473	.800
.289	.485	.825
.291	.496	.850
.293	.504	.875
.293	.505	.900
.290	.491	.925
.278	.438	.950
.246	.294	.975

$p_i = .30$	$p_j = .70$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.210	–.000	.000
.213	.014	.025
.216	.028	.050
.219	.043	.075
.222	.057	.100
.225	.071	.125
.228	.084	.150
.231	.098	.175
.234	.112	.200
.236	.126	.225
.239	.139	.250
.242	.153	.275
.245	.166	.300
.248	.180	.325
.251	.193	.350
.253	.206	.375
.256	.219	.400
.259	.232	.425
.262	.245	.450
.264	.258	.475
.267	.271	.500
.270	.283	.525
.272	.296	.550
.275	.308	.575
.277	.320	.600
.280	.332	.625
.282	.344	.650
.285	.355	.675
.287	.366	.700
.289	.376	.725
.291	.386	.750
.293	.395	.775
.295	.404	.800
.296	.411	.825
.297	.416	.850
.298	.418	.875
.297	.414	.900
.294	.401	.925
.288	.372	.950
.285	.355	.975

$p_i = .30$ $p_j = .80$

p_{ij}	ϕ_{ij}	ρ_{ij}
.240	–.000	.000
.242	.013	.025
.245	.026	.050
.247	.039	.075
.249	.052	.100
.252	.064	.125
.254	.077	.150
.256	.089	.175
.259	.101	.200
.261	.113	.225
.263	.125	.250
.265	.137	.275
.267	.149	.300
.269	.160	.325
.271	.171	.350
.273	.182	.375
.275	.193	.400
.277	.204	.425
.279	.214	.450
.281	.224	.475
.283	.234	.500
.285	.244	.525
.286	.253	.550
.288	.262	.575
.290	.271	.600
.291	.279	.625
.293	.287	.650
.294	.295	.675
.293	.302	.700
.297	.308	.725
.298	.315	.750
.299	.320	.775

$p_i = .30$	$p_j = .90$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.270	–.000	.000
.272	.011	.025
.273	.022	.050
.274	.033	.075
.276	.043	.100
.277	.053	.125
.279	.063	.150
.280	.073	.175
.281	.083	.200
.283	.092	.225
.284	.101	.250
.285	.110	.275
.286	.119	.300
.288	.127	.325
.289	.136	.350
.290	.143	.375
.291	.151	.400
.292	.158	.425
.293	.165	.450
.294	.172	.475
.295	.178	.500
.295	.184	.525
.296	.190	.550
.297	.195	.575
.298	.200	.600
.298	.205	.625
.299	.209	.650
.299	.213	.675

$p_i = .40$ $p_j = .40$

p_{ij}	ϕ_{ij}	ρ_{ij}
.160	–.000	.000
.164	.015	.025
.167	.031	.050
.171	.047	.075
.175	.062	.100
.179	.078	.125
.183	.094	.150
.186	.110	.175
.190	.126	.200
.194	.142	.225
.198	.158	.250
.202	.174	.275
.206	.191	.300
.210	.207	.325
.214	.224	.350
.218	.241	.375
.222	.258	.400
.226	.276	.425
.230	.293	.450
.235	.311	.475
.239	.329	.500
.243	.348	.525
.248	.367	.550
.253	.386	.575
.257	.406	.600
.262	.426	.625
.267	.446	.650
.272	.468	.675
.278	.490	.700
.283	.512	.725
.289	.536	.750
.295	.561	.775
.301	.587	.800
.308	.615	.825
.315	.645	.850
.323	.678	.875
.333	.719	.900
.346	.776	.925
.373	.886	.950

$p_i = .40$	$p_j = .50$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.200	–.000	.000
.204	.016	.025
.208	.031	.050
.212	.047	.075
.215	.063	.100
.219	.079	.125
.223	.095	.150
.227	.110	.175
.231	.126	.200
.235	.142	.225
.239	.159	.250
.243	.175	.275
.247	.191	.300
.251	.208	.325
.255	.224	.350
.259	.241	.375
.263	.258	.400
.267	.275	.425
.272	.293	.450
.276	.310	.475
.280	.328	.500
.285	.346	.525
.289	.365	.550
.294	.383	.575
.299	.402	.600
.303	.422	.625
.308	.442	.650
.313	.462	.675
.318	.483	.700
.324	.505	.725
.329	.527	.750
.335	.551	.775
.341	.575	.800
.347	.600	.825
.354	.628	.850
.361	.658	.875
.370	.692	.900
.380	.736	.925
.397	.803	.950

$p_i = .40$ $p_j = .60$

p_{ij}	ϕ_{ij}	ρ_{ij}
.240	–.000	.000
.244	.015	.025
.247	.031	.050
.251	.046	.075
.255	.062	.100
.259	.077	.125
.262	.093	.150
.266	.108	.175
.270	.124	.200
.274	.140	.225
.277	.155	.250
.281	.171	.275
.285	.187	.300
.289	.203	.325
.293	.219	.350
.296	.235	.375
.300	.251	.400
.304	.267	.425
.308	.284	.450
.312	.300	.475
.316	.317	.500
.320	.334	.525
.324	.351	.550
.328	.368	.575
.333	.386	.600
.337	.404	.625
.341	.422	.650
.346	.440	.675
.350	.459	.700
.355	.477	.725
.359	.496	.750
.364	.516	.775
.368	.535	.800
.373	.555	.825
.378	.574	.850
.382	.593	.875
.386	.608	.900
.387	.612	.925
.378	.573	.950
.326	.359	.975

$p_i = .40$	$p_j = .70$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.280	–.000	.000
.283	.015	.025
.287	.030	.050
.290	.044	.075
.293	.059	.100
.297	.074	.125
.300	.089	.150
.303	.104	.175
.307	.118	.200
.310	.133	.225
.313	.148	.250
.316	.163	.275
.320	.177	.300
.323	.192	.325
.326	.207	.350
.330	.222	.375
.333	.236	.400
.336	.251	.425
.340	.266	.450
.376	.281	.475
.346	.296	.500
.350	.311	.525
.353	.326	.550
.357	.341	.575
.360	.356	.600
.363	.371	.625
.367	.386	.650
.370	.401	.675
.373	.416	.700
.377	.430	.725
.380	.445	.750
.383	.459	.775
.386	.472	.800
.389	.483	.825
.391	.493	.850
.391	.497	.875
.390	.489	.900
.382	.456	.925
.361	.361	.950
.307	.122	.975

$p_i = .40$ $p_j = .80$

p_{ij}	ϕ_{ij}	ρ_{ij}
.320	–.000	.000
.323	.013	.025
.325	.027	.050
.328	.041	.075
.331	.054	.100
.333	.068	.125
.336	.081	.150
.339	.095	.175
.341	.108	.200
.344	.121	.225
.346	.134	.250
.349	.147	.275
.351	.161	.300
.354	.173	.325
.357	.186	.350
.359	.199	.375
.362	.212	.400
.364	.225	.425
.366	.237	.450
.369	.249	.475
.371	.262	.500
.374	.274	.525
.376	.286	.550
.378	.298	.575
.381	.309	.600
.383	.321	.625
.385	.332	.650
.387	.342	.675
.389	.353	.700
.391	.363	.725
.393	.372	.750
.395	.380	.775
.396	.387	.800
.397	.393	.825
.397	.395	.850
.397	.392	.875
.394	.379	.900
.389	.350	.925
.379	.300	.950
.374	.274	.975

$p_i = .40$	$p_j = .90$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.360	.000	.000
.362	.012	.025
.363	.023	.050
.365	.034	.075
.367	.046	.100
.368	.057	.125
.370	.068	.150
.372	.079	.175
.373	.089	.200
.375	.100	.225
.376	.110	.250
.378	.121	.275
.379	.131	.300
.381	.141	.325
.382	.151	.350
.384	.160	.375
.385	.170	.400
.386	.179	.425
.388	.188	.450
.389	.197	.475
.390	.205	.500
.391	.213	.525
.393	.221	.550
.394	.229	.575
.395	.236	.600
.396	.243	.625
.397	.250	.650
.398	.256	.675
.399	.263	.700

$p_i = .50$	$p_j = .50$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.250	–.000	.000
.254	.016	.025
.258	.032	.050
.262	.048	.075
.266	.064	.100
.270	.080	.125
.274	.096	.150
.278	.112	.175
.282	.128	.200
.286	.144	.225
.290	.161	.250
.294	.177	.275
.298	.194	.300
.303	.210	.325
.307	.227	.350
.311	.244	.375
.315	.262	.400
.320	.279	.425
.324	.297	.450
.329	.315	.475
.333	.333	.500
.338	.352	.525
.343	.370	.550
.347	.390	.575
.352	.409	.600
.357	.429	.625
.363	.450	.650
.368	.471	.675
.373	.493	.700
.379	.516	.725
.385	.539	.750
.391	.564	.775
.398	.590	.800
.405	.618	.825
.412	.649	.850
.421	.685	.875
.433	.731	.900
.451	.804	.925
.490	.961	.950

$p_i = .50$	$p_j = .60$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.300	–.000	.000
.304	.016	.025
.308	.031	.050
.312	.047	.075
.315	.063	.100
.319	.079	.125
.323	.094	.150
.327	.110	.175
.331	.126	.200
.335	.142	.225
.339	.159	.250
.343	.175	.275
.347	.191	.300
.351	.208	.325
.355	.224	.350
.359	.241	.375
.363	.258	.400
.367	.275	.425
.372	.293	.450
.376	.310	.475
.380	.328	.500
.385	.346	.525
.389	.364	.550
.394	.383	.575
.399	.402	.600
.403	.422	.625
.408	.442	.650
.413	.462	.675
.418	.483	.700
.424	.505	.725
.429	.527	.750
.435	.551	.775
.441	.575	.800
.447	.601	.825
.454	.629	.850
.462	.661	.875
.472	.700	.900
.485	.756	.925

$p_i = .50$ $p_j = .70$

p_{ij}	ϕ_{ij}	ρ_{ij}
.350	–.000	.000
.353	.015	.025
.357	.030	.050
.360	.045	.075
.364	.060	.100
.367	.076	.125
.371	.091	.150
.374	.106	.175
.378	.121	.200
.381	.137	.225
.385	.152	.250
.388	.168	.275
.392	.183	.300
.396	.199	.325
.399	.215	.350
.403	.231	.375
.407	.247	.400
.410	.263	.425
.414	.279	.450
.418	.296	.475
.422	.312	.500
.425	.329	.525
.429	.346	.550
.433	.363	.575
.437	.380	.600
.441	.398	.625
.445	.416	.650
.449	.434	.675
.454	.452	.700
.458	.471	.725
.462	.490	.750
.467	.508	.775
.471	.527	.800
.475	.546	.825
.479	.564	.850
.483	.580	.875
.485	.590	.900
.483	.580	.925
.465	.501	.950
.389	.171	.975

$p_i = .50$	$p_j = .80$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.400	–.000	.000
.403	.014	.025
.406	.028	.050
.408	.042	.075
.411	.056	.100
.414	.070	.125
.417	.084	.150
.420	.098	.175
.422	.112	.200
.425	.126	.225
.428	.140	.250
.431	.154	.275
.434	.168	.300
.436	.182	.325
.439	.196	.350
.442	.211	.375
.445	.225	.400
.448	.239	.425
.451	.253	.450
.454	.268	.475
.456	.282	.500
.459	.296	.525
.462	.311	.550
.465	.325	.575
.468	.340	.600
.471	.354	.625
.474	.368	.650
.476	.382	.675
.479	.396	.700
.482	.409	.725
.484	.422	.750
.487	.435	.775
.489	.445	.800
.491	.453	.825
.491	.456	.850
.490	.449	.875
.484	.422	.900
.470	.349	.925
.436	.180	.950
.359	–.204	.975

$p_i = .50$ $p_j = .90$

p_{ij}	ϕ_{ij}	ρ_{ij}
.450	.000	.000
.452	.012	.025
.454	.024	.050
.455	.035	.075
.457	.047	.100
.459	.059	.125
.461	.070	.150
.462	.082	.175
.464	.093	.200
.466	.105	.225
.467	.117	.250
.469	.128	.275
.471	.139	.300
.473	.151	.325
.474	.162	.350
.476	.173	.375
.478	.184	.400
.479	.195	.425
.481	.206	.450
.483	.217	.475
.484	.227	.500
486	.238	.525
.487	.248	.550
.489	.258	.575
.490	.268	.600
.492	.278	.625
.493	.287	.650
.494	.296	.675
.496	.304	.700
.497	.312	.725
.498	.320	.750
.499	.326	.775
.500	.332	.800
.500	.335	.825
.500	.336	.850
.500	.331	.875
.498	.320	.900
.495	.303	.925
.496	.304	.950

$p_i = .60$	$p_j = .60$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.360	–.000	.000
.364	.015	.025
.367	.031	.050
.371	.047	.075
.375	.062	.100
.379	.078	.125
.382	.094	.150
.386	.110	.175
.390	.126	.200
.394	.142	.225
.398	.158	.250
.402	.174	.275
.406	.191	.300
.410	.207	.325
.414	.224	.350
.418	.241	.375
.422	.258	.400
.426	.276	.425
.430	.293	.450
.435	.311	.475
.439	.329	.500
.443	.348	.525
.448	.367	.550
.453	.386	.575
.457	.405	.600
.462	.426	.625
.467	.446	.650
.472	.467	.675
.477	.489	.700
.493	.512	.725
.489	.536	.750
.495	.561	.775
.501	.588	.800
.508	.617	.825
.516	.650	.850
.526	.692	.875
.540	.750	.900
.565	.853	.925

$p_i = .60$ $p_j = .70$

p_{ij}	ϕ_{ij}	ρ_{ij}
.420	–.000	.000
.423	.015	.025
.427	.030	.050
.430	.045	.075
.433	.060	.100
.437	.075	.125
.440	.091	.150
.444	.106	.175
.447	.121	.200
.451	.137	.225
.454	.153	.250
.458	.169	.275
.461	.185	.300
.465	.201	.325
.469	.217	.350
.472	.234	.375
.476	.251	.400
.480	.267	.425
.484	.285	.450
.488	.302	.475
.492	.320	.500
.496	.338	.525
.500	.356	.550
.504	.375	.575
.508	.394	.600
.513	.413	.625
.517	.433	.650
.522	.453	.675
.526	.474	.700
.531	.496	.725
.536	.518	.750
.542	.542	.775
.547	.567	.800
.553	.594	.825
.560	.624	.850
.568	.660	.875
.579	.708	.900
.596	.782	.925

$p_i = .60$	$p_j = .80$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.480	–.000	.000
.483	.013	.025
.485	.027	.050
.488	.041	.075
.491	.055	.100
.494	.070	.125
.496	.084	.150
.499	.098	.175
.502	.113	.200
.505	.127	.225
.508	.142	.250
.511	.157	.275
.514	.171	.300
.517	.186	.325
.520	.202	.350
.523	.217	.375
.526	.232	.400
.529	.248	.425
.532	.264	.450
.535	.280	.475
.538	.296	.500
.541	.312	.525
.544	.328	.550
.548	.345	.575
.551	.362	.600
.554	.379	.625
.558	.396	.650
.561	.413	.675
.564	.431	.700
.568	.448	.725
.571	.466	.750
.575	.483	.775
.578	.500	.800
.581	.516	.825
.584	.529	.850
.585	.535	.875
.583	.525	.900
.572	.469	.925
.535	.282	.950
.429	–.262	.975

$p_i = .60$ $p_j = .90$

p_{ij}	ϕ_{ij}	ρ_{ij}
.540	.000	.000
.542	.012	.025
.543	.023	.050
.545	.035	.075
.547	.047	.100
.549	.059	.125
.550	.071	.150
.552	.083	.175
.554	.095	.200
.556	.108	.225
.558	.120	.250
.559	.132	.275
.561	.145	.300
.563	.157	.325
.565	.170	.350
.567	.182	.375
.569	.195	.400
.571	.208	.425
.572	.220	.450
.574	.233	.475
.576	.246	.500
.578	.258	.525
.580	.271	.550
.582	.283	.575
.583	.296	.600
.585	.308	.625
.587	.320	.650
.589	.332	.675
.590	.343	.700
.592	.353	.725
.593	.362	.750
.594	.369	.775
.595	.373	.800
.592	.370	.825
.592	.356	.850
.587	.320	.875
.576	.246	.900
.555	.099	.925
.515	–.171	.950
.430	–.178	.975

$p_i = .70$	$p_j = .70$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.490	–.000	.000
.493	.014	.025
.496	.029	.050
.499	.043	.075
.502	.058	.100
.505	.073	.125
.508	.088	.150
.512	.103	.175
.515	.118	.200
.518	.134	.225
.521	.149	.250
.525	.165	.275
.528	.181	.300
.531	.197	.325
.535	.214	.350
.538	.230	.375
.542	.247	.400
.545	.264	.425
.549	.282	.450
.553	.299	.475
.557	.317	.500
.560	.336	.525
.564	.354	.550
.568	.374	.575
.573	.393	.600
.577	.413	.625
.581	.434	.650
.586	.455	.675
.590	.478	.700
.595	.501	.725
.600	.525	.750
.606	.551	.775
.612	.580	.800
.619	.612	.825
.627	.652	.850
.638	.706	.875
.656	.793	.900
.691	.959	.925

$p_i = .70$ $p_j = .80$

p_{ij}	ϕ_{ij}	ρ_{ij}
.560	–.001	.000
.562	.013	.025
.565	.026	.050
.567	.040	.075
.570	.054	.100
.572	.068	.125
.575	.082	.150
.578	.096	.175
.580	.111	.200
.583	.125	.225
.586	.140	.250
.588	.155	.275
.591	.170	.300
.594	.186	.325
.597	.202	.350
.600	.217	.375
.603	.234	.400
.606	.250	.425
.609	.267	.450
.612	.284	.475
.615	.301	.500
.618	.318	.525
.622	.336	.550
.625	.355	.575
.628	.373	.600
.632	.393	.625
.636	.412	.650
.639	.432	.675
.643	.453	.700
.647	.475	.725
.651	.497	.750
.656	.521	.775
.660	.547	.800
.666	.576	.825
.672	.610	.850
.680	.653	.875
.691	.715	.900

$p_i = .70$	$p_j = .90$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.630	.000	.000
.632	.011	.025
.633	.023	.050
.635	.034	.075
.636	.046	.100
.638	.058	.125
.640	.070	.150
.641	.083	.175
.643	.095	.200
.645	.108	.225
.647	.121	.250
.648	.134	.275
.650	.147	.300
.652	.160	.325
.654	.174	.350
.656	.187	.375
.658	.201	.400
.660	.215	.425
.662	.230	.450
.664	.244	.475
.666	.259	.500
.668	.273	.525
.670	.288	.550
.672	.303	.575
.674	.318	.600
.676	.333	.625
.678	.349	.650
.680	.364	.675
.682	.378	.700
.684	.393	.725
.686	.406	.750
.687	.417	.775
.688	.424	.800
.688	.425	.825
.687	.411	.850
.681	.371	.875
.667	.271	.900
.636	.041	.925

$p_i = .80$	$p_j = .80$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.640	–.001	.000
.642	.012	.025
.644	.024	.050
.646	.037	.075
.648	.050	.100
.650	.063	.125
.652	.077	.150
.654	.091	.175
.657	.105	.200
.659	.119	.225
.661	.133	.250
.664	.148	.275
.666	.163	.300
.669	.178	.325
.671	.194	.350
.674	.210	.375
.676	.226	.400
.679	.242	.425
.681	.259	.450
.684	.276	.475
.687	.294	.500
.690	.312	.525
.693	.331	.550
.696	.350	.575
.699	.369	.600
.702	.389	.625
.706	.410	.650
.709	.432	.675
.713	.455	.700
.717	.479	.725
.721	.505	.750
.725	.534	.775
.731	.568	.800
.738	.611	.825
.747	.669	.850
.762	.761	.875
.787	.922	.900

$p_i = .80$ p_{ij}	$p_j = .90$ ϕ_{ij}	ρ_{ij}
.720	.000	.000
.721	.011	.025
.723	.021	.050
.724	.032	.075
.725	.044	.100
.727	.055	.125
.728	.067	.150
.729	.079	.175
.731	.091	.200
.732	.104	.225
.734	.117	.250
.736	.130	.275
.737	.144	.300
.739	.158	.325
.741	.172	.350
.742	.186	.375
.744	.201	.400
.746	.216	.425
.748	.232	.450
.750	.247	.475
.752	.264	.500
.754	.280	.525
.756	.297	.550
.758	.314	.575
.760	.332	.600
.762	.350	.625
.764	.369	.650
.767	.388	.675
.769	.408	.700
.771	.429	.725
.774	.451	.750
.777	.474	.775
.780	.499	.800
.783	.527	.825
.787	.560	.850
.792	.599	.875
.797	.643	.900
.801	.675	.925
.792	.603	.950
.692	–.231	.975

$p_i = .90$	$p_j = .90$	
p_{ij}	ϕ_{ij}	ρ_{ij}
.810	.002	.000
.811	.011	.025
.812	.020	.050
.813	.030	.075
.814	.040	.100
.814	.050	.125
.815	.060	.150
.816	.071	.175
.817	.083	.200
.819	.094	.225
.820	.107	.250
.821	.119	.275
.822	.132	.300
.823	.145	.325
.824	.159	.350
.826	.174	.375
.827	.188	.400
.828	.203	.425
.830	.219	.450
.831	.235	.475
.833	.252	.500
.834	.269	.525
.836	.287	.550
.838	.306	.575
.839	.325	.600
.841	.346	.625
.843	.368	.650
.845	.391	.675
.848	.417	.700
.850	.447	.725
.853	.482	.750
.858	.528	.775
.863	.590	.800
.872	.683	.825
.885	.833	.850