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NOTES AND DISCUSSIONS

A Simple Algebraic Development of Horst's Suppressor Variables

In his well-known *SSRC Bulletin* of 1941 on the prediction of personal adjustment, Horst has given a supplementary discussion of the use of what he calls a "suppression variable" in predicting a criterion. The ideal case of this sort of variable is one that is wholly uncorrelated with the criterion but which nevertheless improves the prediction of the criterion. The 'suppressor' achieves this result by the fact of its being correlated with the independent variables. The psychological meaning of this relationship is that no independent variable used in practice is 'pure' for the component which is related to the criterion, so that, in predicting from a given independent variable, we are always predicting components which are irrelevant. The function of a suppressor variable is to 'suppress' these components of the independent variable which are not correlates of the criterion. Horst has pointed out that the usual approach to a prediction problem more or less precludes the employment of such variables, because the original setting up of the multiple prediction equation follows a *selection* of variables in which those potential predictors not having appreciable correlation with the criterion are systematically excluded. It is, of course, impossible to improve upon the fit given by the conventional least squares method, once the general kind of function has been chosen. If the suppression variable is once allowed to appear in the original setting up of the least squares equations, it will necessarily occur in the solution with the optimal weight. The point in the development at which the suppression variable is ordinarily abandoned is at the very beginning, a point at which it is rejected as a candidate because of its lack of association with the criterion. Horst emphasized the fact that he is not deviating in any essential way from the traditional mathematical analysis of a multiple prediction problem, but merely attempting "to indicate more realistic methods of selecting efficient sets of prediction variables."

The logical and psychological discussion given by Horst is straightforward and convincing, although many persons experience a certain amount of bewilderment upon being confronted with the possibility of a variable aiding in the prediction of anything to which it holds no statistical relation. Horst's mathematical treatment of the concept is in terms of matrix algebra formulation which is beyond the comprehension of many psychologists, and furthermore he makes use of a number of simplifying assumptions, such as that all the intercorrelations of independent variables are equal, and that the criterion correlations are either zero or all equal. The writer has found it helpful in discussing the notion of a suppression variable with students to make use of a simple algebraic development with only one independent variable and one suppressor, an exposition which is free of artificial limiting assumptions and which dovetails very easily in its algebraic form with the logical conceptualization of what is happening. It is merely as a pedagogical device that the following treatment is offered.

Suppose it is desired to predict a criterion y from an independent variable x (henceforth referred to as the *predictor*), where the correlation between them is imperfect. Let there be a third variable w, henceforth called the *suppressor*, which is positively correlated with the predictor x but is wholly uncorrelated with the criterion. Expressing these three quantities as absolute deviates from their respective means, define a difference quantity

$$d = x - \lambda w$$

where λ is a weight whose optimal value is to be determined. This difference variable $(x - \lambda w)$ may then be conveniently thought of as a sort of 'corrected' value of the predictor, with some weighted component λw which is associated with the suppressor subtracted out. It is this 'corrected' predictor which we intend to use in predicting the criterion. Then the problem is to minimize the sum of squares in predicting y from d, that is

$$\Sigma(y - \mu d)^2 = Min.$$

the solution of which is, of course, the regression coefficient of y on d, namely

$$\mu = \frac{\Sigma dy}{\Sigma d^2}$$

The sum of squares when thus minimized is then simply

$$SS_R = \Sigma y^2 - \frac{(\Sigma dy)^2}{\Sigma d^2} = Min.$$

Now this error is a minimum when λ is so chosen that the subtracted term is a maximum, since Σy^2 is not a function of λ. The cross-product Σdy in the numerator of the subtracted term is also independent of the value chosen for λ, since it expands to

$$\Sigma dy = \Sigma\left[y(x-\lambda w)\right] = \Sigma xy - \lambda\Sigma yw$$
$$= \Sigma xy$$

since, by hypothesis, $\Sigma yw = 0$, the suppressor being uncorrelated with the criterion.

Consequently the sum of squares is minimized when the denominator of the subtracted term is a minimum, that is, when λ is so chosen that

$$\Sigma d^2 = \Sigma(x-\lambda w)^2 = Min.$$

But this means that the optimal value of λ is the regression coefficient of the predictor upon the suppressor. In terms of the common sense of the situation, this gives the student closure because he sees that we are essentially predicting the criterion y from the *residuals* of the predictor upon the suppressor, i.e., from "that part of the predictor which is not associated with the suppressor." Since the suppressor is not associated with the criterion, this is exactly the "part" of x with which we want to predict. This kind of thinking is already available to the student from his study of simple partial correlation. A substitution of $r_{yw} = 0$ in the usual formulas for the multiple regression weights of y on x and w will furnish additional satisfaction to the student, when he sees that the weights resulting from such substitution are the same ones he gets by expanding $y = \mu(x - \lambda w)$ in terms of the values of λ and μ just determined.

It is sometimes advanced by a puzzled individual that if such a state of affairs is possible, it ought therefore to be possible to *perfect* prediction by including a great number of variables which are independent of the criterion—"partialling out the whole remainder of the universe," so to speak. The theoretical possibility of this procedure is admitted, but the unlikelihood of being able to locate and adequately to measure all of the irrelevant components of the predictor can be indicated. If all of such components *should* be found, what would be left would be 'pure' for the criterion, and hence could predict it perfectly, provided also that the criterion itself is 'pure' for the component in question. This latter assumption would of course never be fulfilled in any empirical case.

What amount of improvement occurs in this simple case? Taking the present case of two independent variables, the squared correlation without the suppressor is

$$r_1^2 = \frac{SS_Y}{SS_y} = \frac{(\Sigma xy)^2}{\Sigma x^2 \Sigma y^2}\ .$$

The square of the new correlation, making use of the suppressor, is

$$r_2^2 = \frac{SS_{Y_2}}{SS_y} = \frac{(\Sigma dy)^2}{\Sigma d^2 \Sigma y^2} = \frac{(\Sigma xy)^2}{\Sigma d^2 \Sigma y^2} \,.$$

The ratio of the two coefficients of determination is then

$$\frac{r_2^2}{r_1^2} = \frac{\Sigma x^2}{\Sigma d^2} \frac{\Sigma x^2}{\Sigma (x - \lambda w)^2} = \frac{s_x^2}{s_x^2 (1 - r_{xw}^2)} = \frac{1}{1 - r_{xw}^2} \,.$$

Taking the square root of both sides,

$$\frac{r_2}{r_1} = \frac{1}{\sqrt{1 - r_{xw}^2}}$$

Thus, the ratio of the correlation, using the suppressor to the original correlation using the predictor only, equals the reciprocal of the predictor-suppressor alienation coefficient. Suppose that the original correlation of the predictor with the criterion is .45, and suppose that the suppressor correlates .60 with the predictor but is unrelated to the criterion. Then the effect of including the suppressor in the prediction equation is to raise the correlation from .45 to .56. Our ability to predict *y* has risen from .45 to .56 as a result of including *w.*

While it is mathematically possible for the suppressor to account for *all* of the variance of the predictor except that component which is associated with the criterion, anything approaching this situation is of course hardly likely to arise in psychological data. In the numerical example just mentioned, the upper limit on r_{xw} under the condition that r_{xy} = .45 and r_{yw} =.00 is at .89. This upper algebraic limit could be attained only if the 'impurity' were on the side of the predictor only, i.e., if the criterion variable were a function of no variables other than the component of *x* which is independent of *w.* Assuming this upper limit to have actually been attained, the use of the suppressor can be seen to raise the correlation to 1.00, for *w* has by hypothesis a complete overlap with everything in the predictor except that part which predicts (completely) the criterion.

The search for plausible suppressors in any particular case will presumably have to be undertaken from non-statistical considerations of a logical and psychological character. It seems to the writer that this search ought to be particularly rewarding in the case of question-answer personality tests, where there are often many factors which go into the determination of such verbal responses other than those which these devices are avowedly designed to measure. 'Pure cases' of true suppressor variables do not abound in practice, because there are so few variables in psychological data which are completely uncorrelated. As approximations to the pure case we may mention the "correlation" scales which appear in the Minnesota Multiphasic Personality Inventory (Hathaway & McKinley, 1940, 1942), in which items are included on certain scales not because they differentiate criterion groups proper, but because they act to 'suppress' tendencies to response which interfere with optimal discrimination. As yet unpublished studies on highly generalized correction scales in the Multiphasic item pool exemplify the notion of a suppressor variable, although these variables do not occur in the form of actual multiple regression equations (Meehl, [1945]). The underlying reasoning, however, is the same. It seems plausible that a more systematic search for suppressor variables in many areas (e.g., educational prognosis) might be quite profitable, such as the removal of such factors as reading-speed from a group test in the prediction of achievement which involves the capacities which the test samples but does not involve reading. The first step required for such systematic searching is, however, the general acceptance of the notion that a variable may contribute materially to prediction even though it is wholly uncorrelated with the criterion predicted; and the aim of the present treatment has been merely to present this notion in a simple and pedagogically effective form.

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